# Physical Space is a Discrete-Continuous Dual Space of Varying Connectivity Dimensionality Field that Transcends Variable-Based Mathematics

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**Abstract:** We show that developing a Theory of Everything (TOE) to unify all physical interactions requires a spacetime model having: (i) a discrete-continuous dual structure in which physical properties that could hypothetically vary continuously in some abstract sense are discretized upon measurement and (ii) a variable connectivity dimensionality field. Because this type of space transcends variable-based mathematics, we prove a TOE cannot be developed using only differential geometry and other variable-based mathematics. This completely rules out all forms of hidden variable theories. We disprove the holographic principle that posits all information contained in a volume of physical space is encoded on its boundary. Finally, we show how the variable connectivity dimensionality field gives rise to cross-dimensional projections between microstates that leads to the Second Law of Thermodynamics governing Nature's irreversibility. We further show cross-dimensional projections are one mechanism for gauge invariance breaking. Finally, we postulate that electromagnetic fields arise from spacetime gradients in the average connectivity dimensionality deviation.

## 1. Introduction

Although people have observed the universe for millennia, the precise structure of physical space is still unknown. First, there is still debate whether physical space should be modeled as a continuous manifold, a discrete space, or a hybrid between the two at the smallest length scales.(Kempf, 2009; Requardt, 2006) Second, the number of independent spacetime dimensions for observations at extremely small length scales is still unknown.(Dienes *et al.*, 1998; Giudice *et al.*, 1999) Third, there is still debate regarding the ultimate cause for Nature's irreversibility and how to accurately measure it. (Gold, 1962; Feng and Crooks, 2008; Tuisku *et al.*, 2009; Parrondo *et al.*, 2009) Answering these three questions is critical to developing a TOE that will provide a unified theory of physical interactions. Here we show physical space is a type of hybrid (called a discrete-continuous dual space) between a discrete space and a continuous space. Because a non-uniform discrete-continuous dual space has inherent uncertainty in its connectivity dimensionality field,(Manz, 2008) it follows that the number of small-scale spacetime dimensions varies as a function of position in physical space. We show the variable connectivity dimensionality field of physical space gives rise to cross-dimensional projections, gauge invariance breaking, and the Second Law of Thermodynamics that govern Nature's irreversibility.

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## 2. All Physical Properties are Either Discrete or Discrete-Continuous Dual

Processes by which we observe the universe are called physical measurements. People often rely on instruments to aid the five senses for performing physical measurements. For example, microscopes allow us to see things that are very small, and telescopes allow us to see things that are very far away. A closer investigation shows that all physical measurements involve counting. Sometimes this counting is done deliberately, while other times we are hardly aware of it. When we taste something, sensors in our mouths detect molecules and send

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signals through nerves to the brain. These signals are transported by atoms and electrical impulses. Although we may not be familiar with the intricate details of this process, the brain is performing some counting of electrical impulses to determine how something tastes.

Mathematical measurements are very different from physical measurements. Mathematical measurements start with a set of known information and perform a series of computations to arrive at an answer. For example, suppose a rectangle has sides of lengths 10 units and 5 units. The rectangle's area is computed by multiplying these two lengths: 10 units × 5 units = 50 square units. In this case, the area can be determined exactly.

When we try to physically measure the area of a surface, we must perform some type of counting. A way to measure the area of a thin sheet of plastic is to lay it on top of a piece of graph paper. The plastic's area can then be approximated by counting the number of squares on the graph paper fully or partially covered by the plastic. Since the number of squares on the graph paper is countable, this gives a discrete number that approximates the plastic's area. Since area can theoretically take on a continuous range of values, physical measurements do not yield exact areas. However, we can reduce the uncertainty in the measured area by making the squares on the graph paper smaller. A property that yields discrete values when physically measured but might be able to vary continuously in some abstract hypothetical sense is called a *discrete-continuous dual property*.

Do all physical measurements involve uncertainty? No. One can tell with complete certainty that I have two hands, and there are five digits on each hand. Why is it possible to determine exactly how many hands I have, but it is not possible to physically measure areas exactly? Discrete properties come in increments that can be counted exactly, while continuous properties come in increments that cannot be counted exactly. The number of digits on a hand is an example of a discrete property. In summary, there are exactly two types of physical properties: (a) discrete properties that can be measured exactly because they are countable, and (b) discrete-continuous dual properties that cannot be measured exactly because they are countable approximations of a quantity that could hypothetically vary continuously in some abstract sense. Most importantly, no measurable physical properties are *strictly* continuous.

## 3. All Elementary Physical Properties are Invariant Under Regular Isotopies

In topology, the term regular isotopy refers to the equivalence of manifolds under continuous deformation within the embedding space. (Kauffman, 1990) We can think of a regular isotopy as stretching a space in some arbitrary way. Regular isotopies preserve the form of knots. (Kauffman, 1990) Over sufficiently small observation scales no measurable physical properties are continuous, so the continuous deformation of a regular isotopy does not change any small-scale physical properties. Over large observation scales where the continuum approximation holds, discrete-continuous dual properties appear to be continuous so they are not necessarily invariant to regular isotopies of the long-range continuum. Since large-scale physical properties are ultimately derived from small-scale physical properties, all physical properties are ultimately derived from regular-isotopy-invariant small-scale features of physical space.

## 4. Physical Space Transcends Variable-Based Mathematics

A connected space that resembles a discrete space at the smallest measurable length scales and a continuous space over much larger length scales is called a *discrete-continuous dual space*.(Manz, 2008) Because physical space resembles a continuous space when viewed over large distances, but has only discrete and discrete-continuous dual physical properties, we conclude physical space is a type of discrete-continuous dual space. Previously, Manz showed every non-uniform, non-periodic discrete-continuous dual space contains inherent uncertainty in its connectivity dimensionality field.(Manz, 2008) Physical space is not exactly uniform, because it contains non-uniform temperature gradients, non-uniform gravitational fields, non-uniform electromagnetic fields, etc. Physical space is not periodic, because there are not an infinite number of regularly spaced identical copies of each individual person and other objects in the universe. It necessarily follows that the connectivity dimensionality field of physical space contains inherent uncertainty. In such spaces, the connectivity dimensionality field varies as an approximately smooth function of position and takes on some non-integer values.(Manz, 2008)

Connectivity dimensionality is a measure of the number of the independent directions in a region of space.(Manz, 2008) We can represent these independent directions by independent variables (i.e., coordinates) only if the connectivity dimensionality is a non-negative integer.(Manz, 2008) In variable-based mathematics, the number of independent variables is always some non-negative integer; consequently, discrete-continuous dual spaces in which the connectivity dimensionality changes as a function of position transcend variable-based mathematics.(Manz, 2008) Variable-based mathematics includes differential geometry, calculus, linear algebra, tensor analysis, etc. Because of its inherent uncertainty, the connectivity dimensionality field of physical space changes as a function of position; consequently, physical space transcends variable-based mathematics.

## 5. The Nonholographic Principle

It is well-known that solutions to non-stochastic differential equations are determined by their boundary conditions, and initial conditions are a type of boundary condition. The holographic principle posits that physical properties in a volume of physical space are completely encoded on its (light-like) boundary.(Bousso, 2002) This would be true if properties of physical space were entirely governed by nonstochastic differential equations. Since physical space transcends differential geometry (and other variable-based mathematics), there are no differential equations we could integrate to obtain all the properties in a volume of space from its boundary conditions. Therefore, properties in a volume of physical space *cannot* be completely encoded on its boundary. We call this law of Nature the *nonholographic principle*.

## 6. The Second Law of Thermodynamics and Nature's Irreversibility Arise from Cross-Dimensional Projections between Microstates

The best predictor of future events is past events, but we show here that future events cannot be completely predicted by past events. A conceptual mechanical state in which particles move predictably is called a microstate. A predictor theory would presumably involve some set of variables that are followed over time as Nature evolves. Suppose an object with no forces acting on it is initially moving with velocity (0, 0, 1) in an approximately 3-dimensional region of space. Because of inherent uncertainty in the connectivity dimensionality field, the spatial dimensionality is actually given by $3 + \chi$, where the dimensionality deviation $\chi$ is small in magnitude and varies with position. As the particle moves from A to B, its velocity is projected from a space of dimensionality $3 + \chi_A$ to $3 + \chi_B$. If $\chi_A \neq \chi_B$ this *cross-dimensional projection* is irreversible, because it is not one-to-one and onto. Specifically, multiple points from the higher dimensionality space are projected onto the same point in the lower dimensionality space. If we try to reverse this transformation starting from the point in the lower dimensionality space, it is impossible to decide which point in the higher dimensionality space is the "original" or "correct" one. Therefore, the uncertainty in the connectivity dimensionality field causes the velocity and trajectory of a particle to have inherent uncertainty that is irreversible. Instead of following a single, predictable microstate the particle is buffeted from microstate to microstate.

In quantum mechanics, each particle's collection of accessible microstates is described by a wavefunction. According to the postulates of quantum mechanics, measuring a property of a system forces it into a state compatible with that measurement result. For example, measuring an electron's spin along some axis always yields a result of +½ or -½. Before measurement, microstates leading to either a spin of +½ or -½ are accessible. If the particle's spin is measured and found to be +½, the microstates leading to -½ spin are no longer considered accessible; consequently, measurement causes collapse of the wavefunction. Collapse of the wavefunction is an example of irreversibility. It is worth noting that properties of microstates are not completely measurable, because measuring the value of one property can change the values of other properties. For example, measuring a particle's spin parallel to the x-axis can change its spin parallel to the y- and z-axes.

As previously shown, inherent uncertainty in the average connectivity dimensionality field is inversely proportional to the observation distance.(Manz, 2008) Therefore, observing physical space over sufficiently large distances can produce an average connectivity dimensionality with negligible uncertainty. The term *macrostate* describes a system observed over distances sufficiently large that uncertainty in the average connectivity dimensionality can be neglected. Suppose a system starts in macrostate A and can potentially evolve over some fixed time interval to either macrostate B or macrostate C. To determine whether macrostate B or C is the most likely outcome, we have to consider the underlying microstates connecting A to B and A to C. Informally, if there are more microstates connecting A to B than A to C, then B is the more probable outcome. More precisely, we have to take into account transition probabilities between the microstates so that the more likely outcome will be the one having the most *accessible* microstates leading to it.

As originally described by Boltzmann, the logarithm of the number of accessible microstates ($\Omega$) is called entropy (s):

$$s = k_B \ln(\Omega), \quad (1)$$

where $k_B$ is Boltzmann's constant. Formally, $\Omega$ takes into consideration microstates of the universe as a whole. The Second Law of Thermodynamics states that Nature evolves towards increasing total entropy, which is equivalent to our statement above that it evolves towards macrostates having the largest number of accessible microstates. This leads to Nature's irreversibility. We re-iterate that cross-dimensional projections cause the irreversible transitions between microstates that lead to the increase in total entropy described by the Second Law of Thermodynamics. The Standard Model of Particle Physics and General Relativity cannot describe Nature's irreversibility, because they do not take into consideration cross-dimensional projections.

## 7. No-Go for Both Local and Nonlocal Hidden Variable Theories

Einstein, Podolsky, and Rosen postulated that quantum mechanics may be an incomplete theory if particle motions are governed by some set of hidden variables.(Einstein *et al.*, 1935) Since Bell showed certain classes of local hidden variable theories are incompatible with quantum mechanics,(Bell, 1964) experiments have ruled out local hidden variable theories.(Gill *et al.*, 2002; Aspect *et al.*, 1981; Scheidl *et al.*, 2010; Ansmann *et al.*, 2009) Today, it is widely accepted that either quantum mechanics is not governed by any hidden variables or else those hidden variables must be nonlocal.(Gill *et al.*, 2002; Aspect *et al.*, 1981; Scheidl *et al.*, 2010)

According to the hidden variables postulate, it is possible in principle to construct a completely deterministic theory of Nature governed by some set of local and/or nonlocal hidden variables. Such a universe would evolve along a single microstate governed by the hidden variables. Since the variables are hidden we do not know their precise values. In such case, the universe's entropy equals $k_B\log(W)$, where W is the number of possible values for the hidden variables consistent with previous observations. If we measure the value of one of the hidden variables, it no longer remains hidden, so W decreases. If we make no new measurements, W remains the same. Therefore, in every hidden variable model the entropy of the universe either stays the same or *decreases*. In such a manner, all hidden variable models violate the Second Law of Thermodynamics. Numerous experiments performed over the centuries show the universe evolves towards increasing entropy in accordance with the Second Law of Thermodynamics. The direct consequence of this is that the universe cannot be described by a single microstate and cannot be governed by *any* type of hidden variable theory. It does not matter whether such hidden variable theories are local or nonlocal or of any other type; they are completely ruled out. In short, a completely deterministic theory of Nature necessarily violates the Second Law of Thermodynamics.

No violation of the Second Law of Thermodynamics ensues if the universe's entropy is given by $k_B\log(\Omega)$, where $\Omega$ is the number of accessible microstates and Nature transcends variable-based mathematics. In this case, if we make no measurements the universe will naturally evolve via cross-dimensional projections to the macrostate having the highest probability, which of course corresponds to the macrostate having the largest number of accessible microstates. In other words, the universe will evolve towards increasing total entropy as described by the Second Law of Thermodynamics. If we make a new measurement, some degrees of freedom are removed from $\Omega$ as the wavefunction collapses, but this is compensated by cross-dimensional projections that increase $\Omega$ elsewhere in the universe so that the universe's *total* entropy increases even when we perform measurements. We thus arrive at the astonishing conclusion that experimental evidence of the Second Law of Thermodynamics is definitive proof that Nature transcends all forms of variable-based mathematics.

## 8. The Usefulness and Limitations of Variable-Based Theories for Describing Physical Reality

Let us play the devil's advocate and postulate it is possible to create a set of variable-based theories such that this set together can describe the evolution of the physical universe. Let's denote this set as $\mathbb{Z} = \{A, B, C, D, E...\}$ where A is one variable-based theory, B is another variable-based theory, and so forth. For example, A might be the equations describing a harmonic oscillator, B might be Maxwell's laws of electromagnetism, C might be equations describing the thermodynamic properties and phase changes of water, D might be equations describing the motion of a jet plane, E might be stochastic equations describing price changes in the stock market, F might be equations describing the growth of a specific type of microorganism during fermentation, G might be equations describing changes in the population and migratory patterns of a particular plant or animal species, H might be Riemmanian geometry, etc.

From experience, we know that such variable-based theories are extremely useful for describing Nature's properties. So, it is appropriate to consider both the advantages and limitations of variable-based mathematics. There are apparently four positions to argue: (a) perhaps if we are sufficiently clever we could develop a universal variable-based theory that completely describes Nature, (b) perhaps if we are sufficiently clever we could develop a combination of separate variable-based theories that completely describe Nature, (c) perhaps the universe is completely governed by variables but it is impossible for us to be sufficiently clever to describe them because many of them are hidden, and (d) perhaps the universe transcends variable-based mathematics. This paper takes position (d); however, for sake of completeness I will now attempt to argue for the opposing viewpoints.

Position (a): *Perhaps if we are sufficiently clever we could develop a universal variable-based theory that completely describes Nature.* If this is true, then Nature contains a fixed amount of information. For simplicity, let us call the independent information the 'boundary conditions', and initial conditions are a type of boundary condition. Once these boundary conditions are fixed, the remaining properties of the universe can be calculated using variable-based mathematics; for example, by integrating some differential equations or by solving some algebraic equations. Since the boundary conditions are fixed, unchanging inputs that determine which particular universe we live in, the measurement process cannot change the boundary conditions. Particle transmutations provide strong evidence against position (a). Suppose a single photon is described by $n_1$ independent variables and a hydrogen atom (one electron and one proton to keep things simple) is described by $n_2$ independent variables. Absorption of the photon by the hydrogen atom reduces the number of independent variables from $(n_1 + n_2)$ to just $n_2$ as the electron in the hydrogen atom is excited to a higher energy level. This excited hydrogen atom can now release a series of photons by decaying through intermediate energy levels. Perhaps the hydrogen atom releases four photons during this process to decay back to the ground state. If so, the final state of the system is described by many more than $(n_1 + n_2)$ independent variables. Thus, particle transmutations generate and destroy independent mathematical variables. We could instead postulate that the independent variables describe elementary fields. In such case, the number of independent variables equals the number of independent field components per independent spacetime point times the number of independent spacetime points. Numerous experiments show the volume of the universe is increasing which means the number of independent variables in the universe is also increasing. (Such an expansion agrees with and is predicted by the Second Law of Thermodynamics.) This non-conservation of independent variables means there is no way for the final state of the system to be completely encoded in its initial state, no matter how cleverly one tries; thus, position (a) is false.

Position (b): *Perhaps if we are sufficiently clever we could develop a combination of separate variable-based theories that completely describe Nature.* According to this hypothesis, we could describe motions of individual particles by means of variables, and once a particle transmutes we would switch to a new variable-based theory containing new independent variables for those new particles. We also might (or might not) switch to new variables whenever basic characteristics of the system we are studying transmute. For example, a physical chemist might choose new variables when a phase change of matter occurs, a biologist might choose new variables to describe a newly formed species, and an economist might choose new variables to describe a newly formed business. In such a manner, we would attempt to describe Nature by a series of variable-based theories continually adapted to the particular situation. This approach is what we do in practice now, except that it does not completely describe Nature. It is the best we can do, because for reasons described above we cannot use a single variable-based theory to completely describe nature. This description of Nature is necessarily incomplete, because (i) the changing number of independent variables leads to inherent unpredictability and (ii) this approach cannot fully explain cross-dimensional projections between microstates even for non-transmuting particles.

Position (c): *Perhaps the universe is completely governed by variables but it is impossible for us to be sufficiently clever to describe them because many of them are hidden.* This position was disproven in Section 7 above. Entropy is a measure of the number of independent choices for choosing an accessible microstate compatible with an observed macrostate. The increase in entropy described by the Second Law of Thermodynamics means the universe evolves towards an increasing number of accessible microstates. In hidden variable theories, the number of independent variables is fixed but fewer of these independent variables are hidden as we make measurements leading to a predicted *decrease* in entropy as the universe evolves. This prediction is the opposite of what actually happens in Nature.

Position (d): *Perhaps the universe transcends variable-based mathematics.* This is the correct answer. The transcendence of variable-based mathematics does not mean variable-based mathematics are not useful for describing Nature. Rather, we should recognize their limitations and apply them with a sense of caution and humility. There are many situations where variable-based mathematics work remarkably well, and other cases where they do not. The hypercalculus of discrete-continuous dual spaces should provide insights in cases where variable-based mathematics fails.

## 9. Dimensionality Deviation Field of a Charged Elementary Point-Like Particle

A distinction should be made between two separate causes of dimensionality deviations. First, there is inherent uncertainty in the measured average value of the connectivity dimensionality for a region of space, and this uncertainty scales inversely with the radius of that region.(Manz, 2008) Second, there are long-range changes in the average dimensionality deviation that describe the emergence of elementary particles and their associated interaction fields. These are described by some field equations whose precise form is not completely understood at this time. For length scales larger than an elementary particle core, observable motions are possible suggesting the quasi-background space in which the elementary particle core moves has a dimensionality deviation less than one in magnitude. For length scales smaller than an elementary particle core the concept of observable motion is destroyed, which suggests fluxes inside an elementary particle core occur in full dimensions we cannot see. From this we infer the dimensionality deviation magnitude is approximately one inside a charged elementary particle core.

The average dimensionality deviation is a scalar field, so its rate of change as the universe evolves is also a scalar field. Since first-order derivatives are directional (i.e. non-scalar), scalar second order derivatives (e.g. $\nabla^2$) are the lowest order derivatives with respect to *space*-like (as opposed to time-like) dimensions that can occur in the basic field equation describing average dimensionality deviation changes. Point-like elementary particles can be idealized as stationary states with spherical symmetry, which will occur if

$$\frac{1}{S[n,r]}\frac{\partial}{\partial r}\left(S[n,r]\frac{\partial \chi}{\partial r}\right)=0 \qquad (2)$$

where the surface area for a hypersphere whose volume has dimensionality n and radius r is (Sommerville, 1958)

$$S[n,r]=\frac{2\cdot\pi^{n/2}}{\Gamma[n/2]}\cdot r^{n-1}. \quad (3)$$

Since physical space has approximately three long-range spatial dimensions,

$$S[3+\chi,r]\approx 4\pi r^2+\chi\left(\left.\frac{\partial S[n,r]}{\partial n}\right|_{n=3}\right). \quad (4)$$

For regions of space outside elementary particle cores $|\chi|<<1$, so we can expand the field equations as a perturbation series in powers of $\chi$. Substituting Eq. 4 into Eq. 2, the terms linear in $\chi$ give

$$\frac{1}{4\pi r^2}\frac{\partial}{\partial r}\left(4\pi r^2\frac{\partial\chi}{\partial r}\right)\approx 0. \quad (5)$$

Functions of the form $\chi\approx A/r+C$ are the only spherically symmetric solutions to Eq. 5. From this, we infer the average dimensionality deviation field of a point-like elementary particle decays proportional to 1/r for sufficiently large distances. However, this derivation has ignored the temporal dependence of the field and higher order effects.

## 10. Cross-Dimensional Projections as a Gauge Invariance Breaking Mechanism

Gauge invariances are formulated in terms of symmetry groups and describe the invariance of some physical property or field or Lagrangian with respect to certain types of variable transformations.(Jackson and Okun, 2001) Gauge transformations play a key role in the Standard Model of Particle Physics. Over sufficiently large distances that physical space approximates a continuous space of integer dimensionality, positions in physical space are approximated by continuous coordinates, so variable-based mathematics and gauge invariances emerge. At sufficiently small scales where the continuum approximation of physical space fails *or* when the connectivity dimensionality in the continuum approximation is non-integer, independent positions cannot be described by continuous coordinates so gauge invariances fail. If this turns out to be the actual mechanism for gauge invariance breaking, a TOE candidate constructed using only variable-based theories like differential geometry would not be able to accurately describe it, and a TOE candidate based on the hypercalculus of discrete-continuous dual spaces would be required.

## 11. Transition Between Classical and Quantum Regimes

We now consider how the discrete-continuous duality of physical space gives rise to both quantum physics and classical physics. Over small observation scales uncertainty in the average connectivity dimensionality field is appreciable, (Manz, 2008) which causes cross-dimensional projections that induce uncertainty in a particle's momentum. This causes a particle's motion to be described by a wavefunction rather than a deterministic trajectory. Over large observation scales the uncertainty in the average connectivity dimensionality field becomes negligible,(Manz, 2008) which allows variable-based mathematics to emerge as an appropriate approximation. Under such conditions, classical physics emerges and a particle's motion may be described by a deterministic trajectory. Thus, our theory accounts for both classical and quantum regimes.

## 12. Free Will Transcends Variable-Based Mathematics

Some advanced phenomena like free will and consciousness appear to lie at the interface between classical and quantum physics.(Hameroff and Penrose, 1996) Free will is the ability of an intelligent being to make its own decisions. Some people say our choices are predetermined and free will is just an illusion. Because all forms of hidden variable theories are ruled out, a person's choices cannot be completely predetermined. Others may say there is uncertainty in Nature, but this uncertainty is completely random. Suppose one is given the task of deciding whether or not a sequence of characters is completely random. After a little thought, it becomes obvious that one might be able to prove that a sequence of characters is not random, but one can never prove that a sequence is completely random. Examples of nonrandom sequences include sentences (e.g., "This is not random."). Although a sequence like "qoeirndf;paernalkdnf89" may at first glance appear random, one cannot prove it does not contain an encrypted message, because the number of possible encryption schemes is infinite. Therefore, free will transcends variable-based mathematics in a way that is not necessarily random. Further research is needed to better understand the underlying physics of free will and consciousness.(Hameroff and Penrose, 1996)

The following thought experiment demonstrates that free will transcends variable-based mathematics. Suppose you are given the task of predicting my choice of foods for breakfast over some time interval. You might begin by compiling statistics of what foods I like to eat, whether I normally eat at home or at restaurants, and so forth. After doing this for some time, you might compile a list of foods I have never eaten, those which I commonly eat, and those I occasionally eat. You may further discover underlying correlations. For example, cereal and milk are often eaten together. Pancakes and syrup are often eaten together. All of these things may tell you some trends about my choices for breakfast food. They suggest useful correlations but they do not have causative power. The causative power rests with my free will, not the variables you are tracking. For example, I could choose to never eat breakfast during the time period you are supposed to make predictions for. Alternatively, I could choose to only eat foods that I had never previously eaten. These would cause your predictor model (which was based on my past behavior) to completely fail. Alternatively, I could choose to eat food combinations that are completely mismatched. Then you would change your model to start predicting that my behavior is erratic. At that point, I could start becoming more predictable. Every Monday morning, I could go to the same restaurant and order the same meal. You would then change your model to start predicting that I would do so next Monday. However, I could develop a food allergy which would cause me to never eat that meal again. Later, someone might develop a cure for that allergy allowing me to eat that food again. What this shows is that no matter how clever we are, we can't fully predict the outcome of free choices with variables. This is not because we are not sufficiently clever, it is because free will transcends variable-based mathematics.

Since intelligent beings have the power of free will, and free will transcends variable-based mathematics, it follows that only systems transcending variable-based mathematics can support intelligent life. Thus, all universes containing intelligent life transcend variable-based mathematics.

## 13. Postulated Connection between Electromagnetic Fields and Gradients in the Average Dimensionality Deviation

Einstein's General Relativity (GR) theory showed gravitational fields arise from gradients of the spacetime metric.(Einstein, 1920) Cartan's affine theory of spacetime showed particle spin arises from spacetime torsion.(Cartan, 1986) While several postulates have been put forth to explain the origin of electromagnetic interactions, the connection between electromagnetic fields and spacetime structure has remained elusive. Kaluza-Klein theory is the most common ansatz to date for combining electromagnetism with gravity and spacetime structure. In Kaluza-Klein theory, electromagnetism is carried by a fifth spacetime dimension rolled up into a tiny circle. While Kaluza-Klein theory has its advantages, I do not perceive an obvious reason for physical space to contain a fifth dimension rolled up into a tiny circle. I propose that electromagnetic fields arise instead from gradients in the average dimensionality deviation. Section 9 above showed the average dimensionality deviation of a stationary point-like particle decays like $A/r + C$ for sufficiently large r. Both the gravitational and electric potentials of a stationary point-like particle also decay like $A/r + C$ for sufficiently large r. As described above, Einstein established that gravitational fields are due to gradients in the spacetime metric. This leaves a possibility that electromagnetic fields arise from gradients in the average dimensionality deviation, but further testing of this hypothesis is required.

If my hypothesis is true, then for the reasons described in Section 10 above electromagnetic gauge invariance is broken for sufficiently large $|\chi|$. Since Kaluza-Klein theory strictly obeys electromagnetic gauge invariance, it should be possible to design an experiment to tell whether electromagnetism is described by a strictly gauge invariant theory like Kaluza-Klein theory or by a theory like mine in which electromagnetic gauge invariance only holds for sufficiently low $|\chi|$. Careful attention to details would be required to successfully design and perform an experiment that definitively rules out one of these two theories in favor of the other.

## 14. Conclusions

In summary, we showed that Nature cannot be completely described by any variable-based mathematics. This completely rules out all possible types of hidden variable theories. Instead, Nature is described by a discrete-continuous dual space having variable connectivity dimensionality field. Inherent uncertainty in this connectivity dimensionality field becomes larger over small observation scales and negligible over sufficiently large observation scales,(Manz, 2008) and we showed how this causes the transition between the classical regime and the quantum regime. We have shown how this variable connectivity dimensionality field gives rise to cross-dimensional projections that cause the transitions between microstates underlying Nature’s irreversibility embodied in the Second Law of Thermodynamics. The variable connectivity dimensionality field and associated cross-dimensional projections also give rise to gauge invariance breaking and the nonholographic principle. We also showed that all physical properties are either discrete or discrete-continuous dual and arise from small-scale features of physical space that are invariant under regular isotopies. Consequently, a Theory of Everything that unifies all physical interactions must be based on the hypercalculus of discrete-continuous dual spaces having variable connectivity dimensionality field. Finally, I introduced the postulate that electromagnetic fields arise from gradients in the average dimensionality deviation.

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