



# Founding Principles of Space Mixing Theory

**Thomas A. Manz**

26-A Country Squire Court, West Lafayette, IN 47906 email:tmanz@physics.purdue.edu

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**Abstract:** This article develops from first principles some of the key ingredients necessary to successfully construct a Theory of Everything (TOE). The heart of a successful TOE must contain an accurate model of space. The basic principles governing existence have been used to show that space is a network of linked perceptions. The exchange of perceptions, or information, in space is called space mixing. By implicitly representing all the properties of space, this network of linked perceptions has been shown to be a self-scalar field called the Latent Scalar,  $L$ . It is shown that all physical properties of the multiverse are embedded in this self-scalar field, a principle called the *space mixing theorem*. It is also shown that distances between points in a self-scalar field are characterized by an invariant distance parameter and an associated metric tensor, a principle called the *space metric theorem*. Space Mixing Theory is based on a combination of mathematical and logical inferences.

**Keywords:** theory of everything, TOE, unification physics, space-time, multiverse, space mixing theory, philosophy of existence, philosophy of physics

## 1. INTRODUCTION

From antiquity to modern times, people have tried to develop a unified scientific theory that explains the laws governing the world around us. Some of the ancient Greeks believed the world was composed of four elements: earth, water, air, and fire. A prevailing question among ancient Greek philosophy was whether nature behaved as a continuum or as a collection of atoms suspended in total void. (Haubold and Mathai 1998)

A mechanical model of nature was developed during the Scientific Revolution. According to this model, forces acting at a distance moved objects along fixed trajectories described by Newtonian mechanics. At the time of Maxwell, the notion of forces acting at a distance was replaced by the notion of field propagation through the ether. (Jammer 1995)

At the dawn of the 20<sup>th</sup> century, ether theory fell into disrepute when it predicted reference frame dependence of the velocity of light, a prediction contradicted by the Michelson-Morley experiment and Einstein's relativity theory. (Einstein 1920) The subsequent development of quantum mechanics led to unresolved differences with relativity theory on the nature of determinism and causation. (Gribbin 1996, Callender and Huggett 2001)

After the development of relativity theory and quantum mechanics, several new types of particles and interactions were discovered, leading to the Standard Model of particle physics. (Cottingham and Greenwood 1999) However, there still remain a number of physical processes that are not explained by the Standard Model. Among other things, the Standard Model does not provide a unification of gravitation with electromagnetism. Nor does it provide a method for quantizing gravity, i.e. predicting the masses of elementary particles. (Callender and Huggett 2001)

Early efforts to unify electromagnetism and gravity were made by Einstein, Kaluza, and Klein. More recently, several theories have emerged which attempt to explain the quantization of gravity. These methods include string and M theories (Kaku 1999, 2000, Greene 2000), loop quantum gravity (Smolin 2000, Gambini and Pullin 2000), twistor theory (Penrose and Rindler 1987ab), noncommutative geometry (Connes 1994, Madore 1999), chaotic quantization (Beck 2002), simplicial triangulation and lattice models (Smolin 2003, Hamber and Williams 1999), resonance theory (Pinnow 2000), causal set models (Smolin 2003, Markopoulou 2000), and lump models (Requardt 2000). This list includes only a representation of works written on these theories and is not meant to be complete.

A Theory of Everything must also shed light on questions of broad importance to humanity. In particular, it must explain whether supernatural forces exist and if so how they interact with natural forces. I have no personal doubts as to what the answer to this question is, but I recognize the basic reality that many people believe in God whereas many other people do not.

Our world is not wholly characterized by mechanical interactions but involves processes of thought, behavior, and social interactions. In general, a Theory of Everything must contain a broader concept of interaction that extends beyond the four fundamental interactions of electromagnetism, gravitation, and nuclear interactions. It should be adept at explaining abstract thought processes as well as physical interactions.

In summary, it remains a great mystery of how to construct a theory that successfully unifies all interactions. Since 1995, the author has studied this question in great detail. The results of this study have been the emergence of a new theory of physics called Space Mixing Theory, which will be published in a series of articles. This article, the first in the series, explains how the relationship between space and existence can be used to construct a model of space.

## **2. THE MULTIVERSE**

The collection of all space is called the multiverse. Why is it necessary to distinguish between the terms multiverse and universe? A universe is defined as any collection of galaxies and intergalactic objects arising from a common Big Bang explosion. In other words, when energy explodes in a Big Bang, the resulting entity is called a universe. A universe has a beginning time at which the Big Bang occurred, and it also has a radius representing how far the matter has traveled across space since the Big Bang occurred. Our Universe is capitalized to distinguish it from any other universes that may exist. All universes are contained in space and are part of the multiverse. Any universe besides our own would have to be some tens of billions of light-years away from us, beyond the edge of our Universe. See Rees (2003) for a discussion of ideas related to the multiverse.

During the study, it was necessary to develop a formal mathematical framework for deriving physical laws. This was done by starting with two indisputable properties of the multiverse:

- (1) The multiverse materially exists.
- (2) The multiverse contains a very large number of interesting properties.

These two statements are not assumptions upon which the theory is based because they are indisputable truths. (If the multiverse did not materially exist, then materials such as paper could not exist. Thus, statement 1 is proved. Further if the multiverse did not contain a very large number of interesting properties, then people could not exist, for people are interesting and their properties are interesting. Thus, statement 2 is proved.)

From statements (1) and (2) above, it follows the multiverse is a particular set within the class of sets containing interesting properties capable of material existence. Any laws governing every set in this class will also govern the multiverse since it belongs to this class.

My initial representation of the multiverse is that it may be characterized by a set of properties. Example properties of the multiverse include: (1) The multiverse contains the Universe, (2) The Universe contains life, (3) The Universe contains the earth, (4) The Universe contains atoms, (5) Oxygen is heavier than hydrogen, (6) The Civil War ended in 1865, (7) Abraham Lincoln was the 16<sup>th</sup> President of the United States, (8) Z is the last letter of the English alphabet, (9) Gravity causes objects to fall towards the earth, and (10) According to Coulomb's Law, two stationary negative charges repel each other. If one were to write down all such properties of the multiverse, one would in effect have completely defined the multiverse.

A system of formal logic is a set of consistent rules which govern relationships between elements in a set. We encounter many examples of formal logic in our everyday lives. The rules of mathematics, of chess, of exchanging money, and of telecommunications are examples of logical systems. More importantly, the multiverse itself is a logical system. This can be proved as follows:

*The multiverse may be characterized by the set of all its properties that exist. Let us denote this set as  $\{X_\alpha\}$ . Suppose there exists some particular relationship  $R_1$  between two properties  $X_\beta$  and  $X_\gamma$  in this set. Because this relationship is a property of the multiverse, it follows  $R_1 \in \{X_\alpha\}$ . Corresponding to every such possible particular relationship  $R_1$  is a hypothetical mutually exclusive relationship  $notR_1$ . That is,  $R_1$  and  $notR_1$  are defined as two properties such that if  $R_1$  is a property of the multiverse, then  $notR_1$  cannot be a property of the multiverse. It follows the statement  $R_1 \in \{X_\alpha\}$  implies  $notR_1 \notin \{X_\alpha\}$ . A contradiction in a set  $S$  occurs if and only if there exist two particular relationships  $R_1$  and  $R_2 = notR_1$  such that  $R_1 \in S$  and  $R_2 \in S$ . However, by the preceding two statements if  $R_1 \in \{X_\alpha\}$  then  $R_2 = notR_1 \notin \{X_\alpha\}$ . Therefore the multiverse contains no inherent contradictions and is a logical system.*

An example of an illogical system is the following: A boy can have only one biological father. John is the boy's biological father. Joe is the same boy's biological father. John and Joe are two different people.

Understanding the multiverse is a logical system is central to understanding everything in this paper. In formal logic systems, abstract ideas, words, and properties are exact mathematical constructs. Mathematics is the natural language of logical systems. As a logical system, the multiverse is governed by a consistent set of mathematical rules. The goal of a Theory of Everything is to describe what these rules are. This paper uses formal logic to derive important properties of space. These properties tell us how a unified theory of physics must be constructed in order to completely describe all physical properties.

### 3. THE NATURE OF EXISTENCE

Existence is a very mysterious property. We accept existence as an undeniable part of the world around us, and yet we cannot thoroughly explain it. Nearly everyone is familiar with the chicken and egg paradox. A chicken comes from an egg; an egg comes from a chicken.

Therefore, which came first, the chicken or the egg? If we say the egg came first, then where did the first egg come from? Where was the chicken that laid the first egg?

In physics, we run into similar paradoxes when trying to explain how the Universe was created or what makes it up at the smallest scale. Modern physics teaches us that the Universe was created in a Big Bang and that its smallest constituents are elementary particles. This leads us naturally to ask questions like, “What did the pre-universe look like before the Big Bang occurred?” and “What kinds of structures make up elementary particles?” We are also led to ask questions like, “Is there any space outside the Universe? If so, does it contain other universes?” But perhaps the most interesting of all questions are “Why does the Universe exist at all?” and “Why does it have the properties that it does?”

In order to investigate questions of this nature, we need to have some sort of model for existence. This model needs to explain the necessary and sufficient conditions for existence to occur, the structure of existence, and the different types of existence. There are many philosophical theories which try to explain the nature of existence and the world around us. However, our goal is not to derive a merely philosophical theory. Our goal is to formulate a theory which describes the explicit mathematical properties of existence so we can derive the equations of physics from it.

### 3.1 The Two Types of Existence

The term existence applies to abstract ideas and physical objects. We say that physical objects exist materially while abstract ideas do not. Consider the following list of entities: a flashlight, a tree, a dollar bill, the Universe, a proton, a photon, economics, the story *Moby Dick*, Pythagorean’s Theorem, calculus, humor, and tranquility. The first six items in the list are said to exist materially, or non-abstractly. The last six items in the list exist only abstractly. The simplest test to determine whether or not something exists materially is to ask, “Is this subject to gravity?” If yes, the entity is material and called an object. If no, the entity is abstract and called an idea.

Abstract ideas do not take up space directly. The story *Moby Dick* may be written in a book. The book is a material object and takes up space. However, the story *Moby Dick* does not take up space directly. The book could just as well contain blank pages and it would still take up just as much space. The book is subject to gravity, but the story *Moby Dick* itself is not subject to gravity. Thus the story *Moby Dick* is an abstract idea. A phenomenally interesting aspect of abstract ideas is that they must be stored in a material host. The story *Moby Dick* can be stored in a book, a computer, or a person’s brain. All of these are examples of material hosts.

On the other hand, material objects take up space in the literal sense. If one has a material object like a bowling ball, that bowling ball takes up a certain amount of space—there is no getting around this. The ability of the Universe to exist materially is its most fundamental property and separates it from the set of mere abstract existence.

### 3.2 The Condition of Existence

A necessary and sufficient condition for something to exist is that it must be perceived as existing. There is an old paradox which asks the question: “If a tree falls in the forest and no one is there to see it fall, did it really fall?” If we say yes then there is the question of “How do we know?” Well, we cannot know unless we have observed that the tree has fallen. So in effect, the only valid conclusion we can come to is that a tree cannot fall unless someone or something perceives that it has fallen! Perhaps a leaf on the ground under the tree gets crumpled when the tree falls on top of it. If so, then the leaf perceives the tree fall, and therefore the tree fell.

This basic principle can be proved as follows:

*Let X be any property of the multiverse that exists. Consider the hypothetical removal of property X. Two states are said to be identical if and only if all of their properties are equal. The first state of the multiverse contains property X, whereas the second does not. Therefore, these two states are not identical, and the existence of property X effects the state of the multiverse. In general, property X is said to exist if and only if it effects the state of the multiverse. Because every entity contained in the multiverse is part of the multiverse as a whole, it is sufficient that the existence of property X effect the state of any particular entity contained in the multiverse in order for the overall state of the multiverse to be effected by property X. This is also a necessary condition because the multiverse is the collection of all entities contained within it; therefore, if no entity within the multiverse is effected by property X, then the multiverse as a whole cannot be effected by property X. Any entity whose state is effected by property X is said to perceive property X. Therefore, property X exists if and only if it is perceived as existing by some entity in the multiverse.*

An entity can exist if and only if it is perceived by something as existing. Let's take the idea of Santa Claus, for example. Santa Claus is an abstract idea. However, let us suppose that no one anywhere in the world had ever heard of Santa Claus or ever conceived of the idea of Santa Claus. Then would Santa Claus exist? No, he would not. Do abstract ideas effect the state of the multiverse? Yes. Ideas are used by people to change the world in fundamental ways.

Now let's take the example of a particular proton. This proton exerts some gravitational and electromagnetic forces on other particles in the multiverse. Through these interactions, other particles in the multiverse feel the presence of this proton. Therefore this proton exists. In other words, a physical particle exists if and only if it interacts with some other particles in the multiverse.

### **3.3 The Structure of Existence**

The set of all things that exist is called Everything. A capital E is used to distinguish this set from the word everything, which may take on one of several meanings. The theory describing the laws governing this set is called the Theory of Everything. In the above sections, we have seen how the set Everything can be divided into two parts: abstract ideas and physical objects.

Entities within the set Everything must be linked by perceptions. Let us suppose that object A perceives object B as existing. Then there exists a perception between A and B which we will represent as  $B \rightarrow A$ , read "B is perceived by A." Perhaps B is "Santa Claus" and A is "John," then this relationship means "Santa Claus is perceived by John." The reverse of this situation may not be true. Santa Claus cannot perceive John. This is because Santa Claus is an abstract idea. Abstract ideas are perceived, but they do not perceive.

On the other hand, physical objects both perceive and are perceived. A proton is felt by other particles in the multiverse, but also feels the presence of other particles. So in effect, we have a new way to distinguish between abstract and non-abstract entities: an entity is said to be non-abstract if and only if it can perceive the presence of other non-abstract entities.

This runs contrary to the notion that existence is merely mental perception, or consciousness. There are many different types of perception, some mental and some non-mental. For example, gravitation is a type of non-mental perception. A rock falls to the ground because of the gravitational field of the earth, which the rock clearly perceives and interacts with. The

philosophical argument “I think therefore I am” is incomplete. A rock does not think, but it exists. A rock does perceive. Therefore the rock is a non-abstract, material object.

Non-abstract entities must be arranged in patterns through which closed loops of perceptions can be drawn. To see why this is so, let’s consider different ways of arranging four entities A, B, C, and D. In this example, we assume that A, B, C, and D do not interact with any other objects. That is, there is no object E which interacts with A, B, C, or D.

We can see immediately that some arrangements will work and some will not. Consider, for example, the arrangement  $D \rightarrow C \rightarrow B \rightarrow A$ . In this example, the entity D exists because it is perceived by entity C. Similarly, entity B exists because it is perceived by entity A. However, construct A does not exist because it is not perceived by anything as existing. Therefore, since A does not exist, it clearly cannot perceive B. We have a fundamental contradiction. If A does not perceive B, then B does not exist! Since B does not exist, it cannot perceive C, implying that C does not exist either. Finally, the non-existence of C means that D cannot be perceived either. Thus, the set {A, B, C, and D} arranged in the pattern  $D \rightarrow C \rightarrow B \rightarrow A$  cannot exist at all!

However, there is a possible solution to this problem. Let us suppose the above objects are arranged in the pattern  $D \rightarrow C \rightarrow B \rightarrow A \rightarrow C$ . In this case, no such contradiction occurs. Object A can exist because it is perceived by object C. Object C can exist because it is perceived by object B, which is in turn perceived by object A. Thus, we have a cyclical pattern of perceptions. In this example, D is an abstract idea because it does not perceive any of the non-abstract objects.

In any case, the only way for existence to occur is if a closed loop of perceptions is formed. The collection of all these closed loops is called space. Abstract entities are linked to space in only one direction and not considered a direct component of space. The collection of all space is called the multiverse and may possibly contain more than one universe.

Returning to the above example, we have seen that the objects A, B, and C exist because they are linked to each other via perceptions. (Perceptions and links are subsequently used as interchangeable terms.) Since A, B, and C exist, the links between them must also exist. Let us call these links  $\ell_1$ ,  $\ell_2$ , and  $\ell_3$ . Then the structure of space in the previous example is  $C\ell_1B\ell_2A\ell_3C$ . Since  $\ell_2$  exists, it must be perceived by something as existing. In other words,  $\ell_2$  is connected by links to other parts of space. In this example, A and B are the links between  $\ell_1$ ,  $\ell_2$ , and  $\ell_3$ . Thus, the roles of objects and links are interchangeable in space.

In fact, one needs a network of links only for existence to occur because the objects in the space are themselves links. Consider for example, the structure  $\ell_1\ell_2\ell_3\ell_4\ell_5\ell_1$ . In this structure,  $\ell_2$  is perceived by  $\ell_4$  with the interaction being  $\ell_3$ . Similarly,  $\ell_4$  is the interaction between  $\ell_3$  and  $\ell_5$ . In this example, the interaction between  $\ell_2$  and  $\ell_5$  is represented by the composite link  $\ell_c = \ell_3\ell_4$ .

If we break down each composite link into smaller and smaller links, then eventually we arrive at links which cannot be broken down into smaller links. These smallest links can have exactly one independent physical property. To see why this is so, consider the following proof:

*Suppose a smallest link  $\ell$  has two independent physical properties  $P_1$  and  $P_2$ . That is,  $P_1$  and  $P_2$  are two independent entities that exist within  $\ell$ . In order for these two properties to exist, they have to be linked to other objects in the space. In other words the structure of the space must contain  $\ell_A P_1 \ell_B$ . and  $\ell_C P_2 \ell_D$ , where  $\ell_A, \ell_B, \ell_C$ , and  $\ell_D$  designate arbitrary links. As described above, the roles of objects and links are interchangeable in space. We can thus view  $P_1$  as the link between  $\ell_A$  and  $\ell_B$ , and  $P_2$  as the link between  $\ell_C$  and  $\ell_D$ . Since  $P_1$  and  $P_2$  are two independent properties, they*

*cannot be the same property. In other words,  $P_1 \neq P_2$ . This implies  $P_1$  and  $P_2$  are two different links. The initial premise stated that both  $P_1$  and  $P_2$  exist within a smallest link  $\ell$  which cannot be broken down into any smaller links. However, the fact that  $P_1$  and  $P_2$  are two different links, and independent of each other, requires that link  $\ell$  be composed of at least two independent links. In other words, link  $\ell$  can be separated into smaller links and therefore cannot be a smallest link unless it has exactly one independent physical property. Since all links can be broken down into smallest links, a representation of space can be achieved in which each independent link in the space has exactly one independent physical property.*

This link model of existence forms the basis of the Theory of Everything.

#### 4. WHAT IS SPACE MIXING?

The exchange of perceptions between entities in space is called space mixing. All perceptions must travel between different points in space-time. Because existence cannot occur without the exchange of perceptions, existence could not occur without space mixing. Suppose object B perceives the existence of object A. This requires a perception to travel from object A to object B. Similarly, if object A perceives the existence of object B, then a perception also travels from object B to object A. All perceptions are a form of information transfer; therefore all physical processes are information transfer. The propagation of perceptions through space requires space to be non-static. That is, there exists a dimension of space which measures when perceptions have been exchanged and space mixing has occurred. We normally refer to this dimension as time.

The irreversibility of space mixing causes time to be irreversible. Consider information traveling from New York to Boston. Is there any way to undo this process? Suppose the same information is sent back to New York. Instead of reversing the first process, we have simply created a loop; the information travels first from New York to Boston and then back to New York. Once space has mixed, there is no way to reverse it. This is equivalent to saying there is no way to travel backwards in time and undo something that has happened.

This can be formally proved as follows:

*Any change to a physical system is space mixing. Time is defined as a measure of space mixing such that whenever space mixing occurs, time has passed according to a local observer. Let some change  $Q$  happen to a system. Time has passed during this process. Now, let us undo the change  $Q$  to the system to bring it to its original state. A change in the system has occurred, so again time has passed. The net result is a positive increase in time for a reference frame locally following the process.*

As mentioned in section 2, the multiverse contains a very large number of interesting properties. The word interesting has the formal definition of *containing independent but related perceivable properties*. This implies that some properties of the multiverse must act independently of each other.

Two properties are said to be independent if one of them can be varied in any way without changing the other. For example, the radius and volume of a sphere are not independent properties because there is no way to change the radius without changing the volume of the sphere. On the other hand, the radius and volume of a cylinder are independent properties

because the volume can be changed without changing the radius by varying the height of the cylinder.

Two properties are said to be related if any connection (either abstract or non-abstract) can be made linking the two properties. An example of an abstract connection is a sentence. In the abstract sense, two properties are said to be related if there exists any sentence which can be made mentioning both properties. For example, the word “electrons” by itself is not interesting. On the other hand, the sentence “Electrons have mass” is interesting because it contains two independent properties, electrons and mass, which are related to each other by the verb have. Similarly, the sentence “Dogs are dogs” is not interesting because dogs are not independent of dogs.

In the non-abstract sense, two properties are related to each other if there exists any physical connection, or link, between them. Space mixing relates independent properties of the multiverse to each other via the exchange of perceptions. Therefore, space mixing is the physical mechanism which makes the multiverse interesting. All kinds of properties like mass, charge, momentum, energy, fields, and particles are related to each other by space mixing.

## 5. THE LATENT SCALAR

### 5.1 More on Properties

A property, by definition, is used to designate a variable which may acquire one of several different values. If we have a property value  $X$ , at the very minimum we also have a second available property value *not*  $X$ . For example, consider the variable color and the associated possible color value *blue*. Then there also exists the possible color value *not blue*.

Associated with every property must be a corresponding operation for determining the particular value of that variable. For example, to every object having the property color must be a corresponding operation to determine the particular color of the object. Every property thus comes in the form of a triplet: the property variable, the operation for determining the value of the property, and the particular value of the property. In this discussion, the property variable will be designated by a capital letter like  $A$ . The corresponding operation will be designated by the hated letter  $\hat{A}$ . Finally, the particular property value will be designated by the small letter  $a$ .

There are many different types of properties. Properties can be simple or complex, subjective or not subjective, abstract or non-abstract, and universal or non-universal.

Subjective properties can always be reduced to a set of non-subjective properties. For example, consider the statement “John is a musical genius.” This statement is subjective. However, it can be reduced to non-subjective statements: “John has written 30 original songs. John has his own CD published. John knows how to play five different musical instruments.”

An abstract property is always stored in a material host. An example of an abstract property is that  $2 + 2 = 4$ . A basket containing four apples could store this mathematical law. This mathematical law could also be stored in the form of writing on a piece of paper, or it could be stored as chemicals in a certain part of a person’s brain. No matter how stored, this abstract concept is always stored in a material host. Associated with every abstract property is a decoding function which derives it from the non-abstract properties of the material host. The most obvious example of this is the processes of reading and writing. In writing, abstract concepts are stored in physical objects such as paper and ink. In reading, the observation of physical objects is decoded to reform the abstract concepts.

Universal laws can be either abstract or non-abstract. Gravitation is a universal non-abstract law. A mathematical law, such as Pythagorean’s Theorem, is an example of a universal abstract



law. A simple test to determine whether or not a law is universal is to ask, “Would Martians discover this law if they physically existed?” Clearly, gravity would also act on Mars, so the Martians could discover the law of gravitation. Also, mathematics would be the same on Mars, so the Martians could derive all of the same laws of math that we use. If they were good mathematicians, they would know that the area of a triangle is equal to one-half its base times its height. The rules of chess are examples of non-universal laws. If Martians physically existed, there is no reason to believe they would play chess, and even if they did, they would probably have an entirely different set of rules.

Physical properties are those which depend directly on the space they are embedded in and are non-abstract. Examples of physical properties include charge, mass, electromagnetic fields, gravitational field, and angular momentum.

## 5.2 Derivation of the Latent Scalar

Let  $\{P_i\}$  be the set of all physical properties of the multiverse with corresponding values  $\{p_i\}$ . I want to show that this set of properties can be derived from a more convenient set of properties.

We know from section 3 above that the multiverse can be represented by a collection of smallest links each consisting of exactly one independent physical property. We also know from section 3 above that links in space have a property of adjacency. That is, if space contains the link structure  $\ell_A \ell_B \ell_C \ell_D$  and doesn't contain the structure  $\ell_A \ell_D$ , we say  $\ell_B$  is adjacent to  $\ell_A$  whereas  $\ell_D$  is not. In general, we may ascribe to each smallest link a location property by specifying which other smallest links in space it is adjacent to. Since each smallest link has exactly one independent physical property, this location property must be its only independent physical property.

However, this does not yet reach our goal of developing a mathematical model of space. Suppose we want to describe space by a function. Then we must ask, “What type of mathematical function gives rise to the properties of space described above?” First, the number of degrees of freedom in space is equal to its number of independent physical properties, which is in turn equal to the number of smallest links. In order for a mathematical function to appropriately describe space, the number of degrees of freedom in the mathematical function must equal the number of degrees of freedom in the space it describes. (If the number of degrees of freedom in the mathematical function is less than the number of degrees of freedom in the space it describes, then the space contains more independent physical properties than the function is capable of describing. On the other hand, if the function contains more degrees of freedom than the space it describes, the function contains more independent properties than the space it describes and thus gives rise to properties that do not physically exist.)

In general, the number of degrees of freedom (number of independent values) of a function is equal to the number of constraints which must be specified to fixate the function. For example, a straight line  $y = mx + b$  has two independent properties. These are its slope,  $m$ , and its intercept,  $b$ . Any two constraints imposed on this equation completely specifies a particular straight line. These constraints could be two points through which the line passes or a specification of its slope and intercept.

In the case of a function describing space, the number of constraints which must be imposed to fixate the function is equal to the number of independent physical properties of space, which also equals the number of smallest links. That is, there is exactly one independent value of the function for each smallest link in space. Since to each smallest link there is exactly one

independent location property, it follows that to every independent value of the function there is a corresponding independent location. Specification of the function values at these independent locations fixates the properties of space.

A scalar function is defined as a function yielding one value for each specification of its arguments. In the case of the function describing space, the argument is the location. As shown previously, to each independent location in space there is exactly one independent value of the function describing space. Therefore the function describing space is a scalar function. I call this function L and give it the name Latent Scalar.

What type of property is L? Thus far, no particular constraints have been placed on the range of L. In general, the range of L takes on an infinite number of possible values. The set of real numbers is a suitable basis for representing L because it contains an infinite number of possible values. L cannot be a complex number because complex numbers have two independent properties, a real part and a complex part. However, there is only one independent property per independent value of L as shown previously. Consequently, L may take on the value of any real number.

Now consider any particular physical property  $P_i$  of the multiverse. Suppose that  $P_i$  can be represented as a function of the set of L values according to equation 1.

$$P_i = p_i = \hat{P}_i(\{L\}) \quad (1)$$

Any property which can be represented in this way is said to be embedded in the set  $\{L\}$ . Next, hypothesize that there is some physical property W of the multiverse which is not embedded in  $\{L\}$ . This means w cannot be expressed as a dependent function  $\hat{W}(\{L\})$ . In other words, w is independent of  $\{L\}$ . Accordingly, the number of independent physical properties of the multiverse must be equal to the total number of independent values of L plus one for w. This introduces a fundamental contradiction because it has previously been proved the total number of independent physical properties of the multiverse is exactly equal to the number of independent L values. Therefore no such physical property W exists which cannot be embedded in the set  $\{L\}$ . Without exception, every physical property of the multiverse is embedded in the Latent Scalar,  $\{L\}$ . This principle is called the space mixing theorem.

Let us consider a particular path in space from location X to location Y. Progress along this path can be parameterized by the quantity

$$\beta = \sum \sqrt{(dL)^2} \quad (2)$$

where dL is the differential change in L along the path. Notice that  $\beta$  increases monotonically along a path and is therefore a suitable distance parameter. Along a path from X to Y, changes in L are given by

$$\Delta L = function(\beta) = function\left(\sum \sqrt{(dL)^2}\right). \quad (3)$$

This is the definition of a self-scalar field, as described in the next section.

Suppose  $L_1 = 5.1465$  at some particular location in space. What physical properties can be derived from this value? Recall that perceptions must be present for existence to occur and that perceptions propagate between two locations in space. Therefore, given one isolated L value, no perceptions exist in this set and therefore we cannot determine any measurable physical property. From the standpoint of measurable properties, any isolated value of L is equivalent to any other isolated value of L. It follows that the absolute magnitude of L is not a measurable property and only changes in L give rise to physical properties.

Suppose we are given a second value  $L_2 = 3.789$  at a location near the first. What physical properties can we determine from these two values? According to equations 2 and 3, we can determine the change  $\Delta L = 3.789 - 5.1465 = -1.3575$  and the distance parameter  $\beta = 1.3575$ .

I call L the Latent Scalar because its absolute magnitude cannot be measured and yet changes in it give rise to all physical properties. The word latent means “*present and capable of becoming though not now visible or active.*” (Mish *et al* 1988)

### 5.3 What is a Self-Scalar Field?

“*Field theories applicable to various types of interaction differ by the number of parameters necessary to define the field and by the symmetry character of the field. In a general sense, a field is a physical entity such that each point in space is a degree of freedom.*” (Panofsky and Phillips 1962, p. 1)

For example, a scalar field  $\phi(\{X^i\})$  represents one scalar variable  $\phi$  as a function of the set of location coordinates  $\{X^i\}$ . Covariant vector  $E_j(\{X^i\})$  and tensor  $\tau_{jk}(\{X^i\})$  fields represent the vector  $E_j$  and tensor  $\tau_{jk}$  components as functions of the location coordinates  $\{X^i\}$ . Contravariant vector and tensor fields are indicated by raised indices:  $E^j(\{X^i\})$  and  $\tau^{jk}(\{X^i\})$  represent the vector  $E^j$  and tensor  $\tau^{jk}$  components as functions of the location coordinates  $\{X^i\}$ . The names covariant and contravariant are easy to confuse so I shall simply refer to them by the more convenient names lowered and raised, respectively. Transformation properties of vectors and tensors are discussed in detail in the book by Kay (1988).

Scalar, vector, and tensor fields have transformation properties similar to those of ordinary scalars, vectors, and tensors. Scalar fields are invariant of the choice of coordinate system. That is,

$$\phi(\{X^i\}) = \phi(\{\bar{X}^j\}) \quad (4)$$

where

$$dX^i = \underline{Q}^i_j d\bar{X}^j. \quad (5)$$

describes the transformation law from one coordinate system to another. Notice that the presence or absence of a bar indicates which coordinate system we are referring to. These equations use the convention that a repeated index is summed over all its possible values. Note that

$$\underline{Q}^i_j = \left( \frac{\partial X^i}{\partial \bar{X}^j} \right) \quad (6)$$

and

$$\bar{Q}^j_i = \left( \frac{\partial \bar{X}^j}{\partial X^i} \right). \quad (7)$$

The quantity  $\bar{Q}^j_i$  is called a mixed tensor because it has one raised and one lowered indices.

This compact notation makes it easy to discover and write out the transformation laws for scalar, vector, and tensor fields represented in different coordinate systems. A repeated raised and lowered index cancels itself. A repeated raised and lowered bar also cancels itself. Thus

$$\tau_{ij} = \bar{Q}^m_i \bar{Q}^n_j \tau_{mn} \quad (8)$$

describes the transformation law for a lowered tensor or tensor field represented in two different coordinate systems.

I define a self-scalar field as a scalar field that is a function of its squared variations. That is, any field parameterized by the sum of squared deviations along a path, as in equation 3. For a self-scalar field, the field itself acts as both the independent and dependent coordinates of the space. A field of this type is called purely relational because it is free from dependence on external coordinates. Like a scalar field, the value of a self-scalar field is completely invariant of the coordinate system.

Self-scalar fields have many interesting mathematical properties. One of these properties is that self-scalar fields embed other vector, scalar, and tensor fields. For example, consider any arbitrary self-scalar field  $Z$  represented as a function of continuous coordinates  $\{\xi^i\}$ . The expectation value, or local average, of squared deviations in  $Z$  along a differential path is given by

$$\langle (dZ)^2 \rangle = \left\langle \left( \left( \frac{\partial Z}{\partial \xi^i} \right) d\xi^i \right) \left( \left( \frac{\partial Z}{\partial \xi^j} \right) d\xi^j \right) \right\rangle = \left\langle \left( \frac{\partial Z}{\partial \xi^i} \right) \left( \frac{\partial Z}{\partial \xi^j} \right) \right\rangle d\xi^i d\xi^j. \quad (9)$$

The expectation operation must be performed because values of the scalar  $Z$  are stochastic in nature. Because the coordinates are non-stochastic, the expectation operation can be pulled inside the summation.

Consider the transformation to a new set of continuous coordinates  $\{\bar{\xi}^i\}$ . Equation 9 becomes

$$\langle (dZ)^2 \rangle = \left\langle \left( \frac{\partial Z}{\partial \bar{\xi}^i} \right) \left( \frac{\partial Z}{\partial \bar{\xi}^j} \right) \right\rangle d\bar{\xi}^i d\bar{\xi}^j. \quad (10)$$

Notice that

$$m_{ij} = \left\langle \left( \frac{\partial Z}{\partial \xi^i} \right) \left( \frac{\partial Z}{\partial \xi^j} \right) \right\rangle \quad (11)$$

defines a quantity that transforms according to equation 8 as a lowered tensor field:

$$\begin{aligned} m_{ij} &= \left\langle \left( \frac{\partial Z}{\partial \xi^i} \right) \left( \frac{\partial Z}{\partial \xi^j} \right) \right\rangle = \left\langle \left( \frac{\partial Z}{\partial \bar{\xi}^m} \right) \left( \frac{\partial \bar{\xi}^m}{\partial \xi^i} \right) \left( \frac{\partial Z}{\partial \bar{\xi}^n} \right) \left( \frac{\partial \bar{\xi}^n}{\partial \xi^j} \right) \right\rangle \\ &\dots = \left( \frac{\partial \bar{\xi}^m}{\partial \xi^i} \right) \left( \frac{\partial \bar{\xi}^n}{\partial \xi^j} \right) \left\langle \left( \frac{\partial Z}{\partial \bar{\xi}^m} \right) \left( \frac{\partial Z}{\partial \bar{\xi}^n} \right) \right\rangle = \bar{Q}_i^m \bar{Q}_j^n \underline{m}_{mn} \end{aligned} \quad (12)$$

where

$$\underline{m}_{mn} = \left\langle \left( \frac{\partial Z}{\partial \bar{\xi}^m} \right) \left( \frac{\partial Z}{\partial \bar{\xi}^n} \right) \right\rangle. \quad (13)$$

An invariant distance parameter is then given by

$$d\beta^2 = \langle dZ^2 \rangle = m_{ij} d\xi^i d\xi^j = \underline{m}_{mn} d\bar{\xi}^m d\bar{\xi}^n. \quad (14)$$

This example illustrates the embedment of a tensor field in a self-scalar field. However, it also proves a fundamental theorem: that every self-scalar field has an invariant distance parameter and an associated metric tensor which describes distances between points in the field. This theorem is called the space metric theorem and is important for establishing the connections between Space Mixing Theory and Einstein's theory of General Relativity.

## 6. THE VISUALIZATION OF SPACE

History shows that how we visualize space strongly effects our perception of the world around us. The ancient Greeks had two competing theories of space: atomism and stoicism. Atomists believed space consists of atoms suspending in total void. Stoics believed space is continuously permeated with a fluid-like substance.

Today people adhere to any of several different visualizations of space. Like the atomists, some people may visualize space as elementary particles in empty surroundings. Others, like the stoics, believe in a fluid, or ether, which continuously permeates space. In addition to the ancient Greek traditions, many modern versions of space have also been developed. Most scientists today believe in a model of space that is intermediate between atomism and stoicism. That is, they believe space is a collection of fields, such as electromagnetic and gravitational fields, which transmit forces between elementary particles. In superstring theories, space is represented by ten-dimensional vibrating strings. Related models replace the strings with membranes and other geometric objects.

Remarkably, there is no consensus today of what space actually is, how many dimensions it is made up of, whether it is fundamentally discrete or continuous, and whether it is infinite or finite. Also, there is no fundamental consensus regarding the causal structure and nature of space. Can space transmit instantaneous action-at-a-distance? Does the past determine a unique future? Is space an absolute or relative concept? If one were to ask these questions to different people, one would obtain a variety of answers depending on the person's viewpoint.

It is remarkable that throughout history scientists who studied the nature of space have often developed conflicting models. Newton developed an absolute model of space and time with instantaneous action-at-a-distance and determined outcomes. Maxwell developed the idea of electromagnetic field propagation through the ether with no action-at-a-distance, but retained determined outcomes. Einstein developed the idea of a curved and relative space-time, rejected action-at-a-distance, but retained determined outcomes. Finally, quantum mechanics rejected determinism, reinstated action-at-a-distance in the form of quantum entanglement, and yet does not fully explain the curvature of space-time. Is it any wonder then that we all have so many different ideas about what space actually is?

In order for you to understand and appreciate the value of Space Mixing Theory, you must begin to develop a model of space which might be at variance with how you currently visualize space. I invite you to try this exercise. Start with a blank piece of paper and write upon it only those things which you absolutely know to be true about space. You are not permitted to write upon it anything you believe to be true about space but cannot prove. Now on a second sheet of blank paper write down how you personally visualize space. This sheet of paper is to contain the list of things about space which you believe are probably true but cannot prove.

Space Mixing Theory conceptually stems from a blank piece of paper approach. One begins with a blank paper and writes upon it only those things which can be proved. Then, one takes a second paper and writes down the things which follow from the first paper. This process is repeated as many times as necessary until all laws governing the multiverse have been derived.

The reader may wonder at this point "What is the practical significance of this?" The practical significance of this is that it provides a method for modeling space which is completely derived from first principles. That is, it tells us what type of mathematical manifold must be used to provide an accurate model of space. No longer must we make assumptions about the structure of space.

## 7. HOW DOES SPACE MIXING THEORY COMPARE TO EXISTING UNIFICATION MODELS?

Space Mixing Theory (SMT) is unique among unification models in that it is the only one with a completeness theorem; meaning, it is the only unification model that has a theorem proving its mathematical description of space embeds all physical properties of the multiverse.

To begin with, an accurate unification model must have the right causal structure. Space mixing is a local process because it occurs through adjacent links in space. This means an accurate description of causality should be based on local time, a reference frame which follows the process from location to location. A local reference frame is the only type which actually measures the progress of space mixing at the site of the process. Therefore, a local reference frame is the only accurate measure of causality.

Confusion abounds over the role of time in quantum gravity models. Space Mixing Theory certainly offers an easy and non-conflicting explanation for causality: *In order for A to causally effect B, information must travel from A to B. Time is a measure of space mixing, and so when information travels from A to B, time passes for a reference frame locally following the process. Therefore, if A causally effects B, it must occur in time before B according to a reference frame locally following the process.* I believe there is no essential motivation for trying to invoke a more complicated or cumbersome explanation of the role of time.

The problem of time arises because the roles of time, causality, and observation are different in General Relativity and quantum mechanics. In General Relativity, there is no preferred reference frame for physical observation or measurement of time. In quantum mechanics, time is absolute and observations cause the wavefunction to collapse. Formulating quantum gravity in terms of local observers is one possible solution to this problem. (Markopoulou 2000)

A similarity between SMT and quantum gravity models is that they have noncommutative geometry when viewed over small distances. Noncommutative geometry means that at small distances the location coordinates (x, y, z) cannot be measured independently. (Connes 1994) The noncommutative geometry of SMT can be proved as follows:

*Consider two adjacent smallest links in space  $A = (x_A, y_A, z_A)$  and  $B = (x_B, y_B, z_B)$ . Each of these smallest links embed exactly one independent physical property, L. The change in L between A and B tells us the distance between them, but it does not tell us the direction. Therefore, the coordinates (x, y, z) cannot be independently measured at the scale of a smallest link.*

At least four independent links in space are needed in order to construct a three-dimensional physical object in space, such as a three-dimensional physical coordinate system. The reason for this is easy to understand. First, consider a single location in space. This is a zero dimensional object. If one considers a second location in space, a line between them can be drawn. This creates a one-dimensional object. Similarly, a third location not located on this line defines a plane in space. If one considers a fourth location not situated on this plane, a three dimensional object is formed. Since each independent link in space corresponds to a single independent location, it follows that four independent links in space are needed to construct a three-dimensional physical object such as a three-dimensional physical coordinate system. Similarly, five independent links are necessary to set up a physical coordinate system having three spatial dimensions and one time dimension.

For distances much larger than the size of a smallest link, noncommutative geometry becomes negligible and the coordinates (x, y, z) appear as independent observables. This is

similar to quantum mechanics for which the noncommutivity of momentum and position is important at small scales but unimportant over large distances.

Next, let's consider whether the Latent Scalar is continuous or discontinuous. Suppose  $L$  takes on two simultaneous values  $L_1$  and  $L_2$  at adjacent mathematical points  $P_1$  and  $P_2$  in space. The definition of a continuous function is that  $L_2 = L_1$  for all such adjacent mathematical points in space. The definition of a discontinuous function is that there exists some pair of adjacent points for which  $L_2 \neq L_1$ . Since  $P_1$  and  $P_2$  are adjacent points, by definition the distance between them is zero, i.e.  $\beta = 0$ . Expressing the distance parameter in terms of the Latent Scalar according to equation 2 gives

$$\beta = \sum \sqrt{dL^2} = |L_2 - L_1|. \quad (15)$$

The condition  $\beta = 0$  directly implies  $L_2 = L_1$  which shows that the Latent Scalar is always a continuous function. In this example, it is important to note that  $P_1$  and  $P_2$  are mathematical points and do not refer to adjacent smallest links in space. The distance across a mathematical point is zero by definition.

On the other hand, the distance across a smallest link is equal to

$$\beta = \int_A^B \sqrt{dL^2} \quad (16)$$

where A and B are two adjacent smallest links in space. The Latent Scalar values at each of these smallest links are independent of each other as discussed in section 5 above. Therefore, we cannot constrain them to be equal. Accordingly, the Latent Scalar varies between smallest links A and B and the integral of equation 16 is nonzero. That is, the smallest links are located nonzero distance apart.

Why is there a smallest measurable distance in space when  $L$  behaves as a continuous function? The answer is because although  $L$  behaves continuously, it is stochastic in nature. The fluctuations in  $L$  introduce quantum mechanical uncertainty into space. These fluctuations are necessary for the unification of quantum mechanics and General Relativity. Without them, space would behave completely classical as in General Relativity. The length operator and metric tensor are two very important components of a quantum gravity model. As demonstrated in equations 11 and 14, the metric tensor and length operator are effected by stochastic fluctuations. This causes uncertainty to exist when trying to measure the location of objects in space. The smallest measurable size represents the minimum uncertainty in location and corresponds to the size of a smallest link. Consequently, although  $L$  behaves as a continuous function, distances cannot be measured to a greater precision than the size of a smallest link.

It is important to point out that the continuous coordinates  $\{\xi^i\}$  referred to in equation 9 do not refer to measurable coordinates. Rather,  $\{\xi^i\}$  are the independent mathematical coordinates which specify the value of the self-scalar field. Physically measured distances and coordinate systems are based on the distance parameter  $\beta$ .

What are the major similarities and differences between SMT and current efforts to develop a unification model? String theories (Gribbin 1998) utilize 10, 11, or 26 dimensions. SMT has far fewer independent dimensions than string theories, but also incorporates dimensions in addition to time and three spatial dimensions. The Latent Scalar can be represented as  $L = L(\{\xi^i\})$  where  $i = 0 \dots n$  and  $\xi^0$  acts as a time coordinate. The number of spatial dimensions,  $n$ , is a local parameter in SMT that can depend on location in space. The goal of SMT is similar to that of

string theories, to unify all physical interactions, but the approach is far different. String theories assume a background space, whereas in SMT the space is relational and there is no background.

Loop quantum gravity (LQG), string theories, Pinnow's resonance theory, and Beck's chaotic quantization model start with some of the basic formulations of quantum mechanics and try to modify them to quantize particle masses and other properties. On the other hand, twistor theory, simplicial triangulation and lattice models, causal set models, and lump models build a representation of space from the ground up. They attempt to derive quantum mechanics and General Relativity as results of a quantum gravity theory. SMT also tries to build a representation of space from the ground up.

Requardt (2000) describes a model of space based on structures he calls lumps. He states, *"Our personal working philosophy is that spacetime at the very bottom (i.e. near or below the notorious Planck scale) resembles or can be modeled as an evolving information processing cellular network, consisting of elementary modules (with, typically, simple internal discrete spaces) interacting with each other via dynamic bonds which transfer the elementary pieces of information among the nodes."* *"A typical example is the geometry of lumps envisaged by Menger. Take as lumps the hypothetical infinitesimal grains of space or spacetime which cannot be further resolved (be it in a practical or principle sense). Let them overlap according to a certain rule so that they can interact or exchange information. Draw a node for each such lump and a bond for each two lumps which happen to overlap."* *"A fortiori one can make these mutual overlaps into dynamical variables, i.e. let them change during the course of evolution."* *"This conjectured geometric order we view as a kind of discrete protospacetime carrying metric, causal, and dimensional structures."* *"The modeling of the depth structure of spacetime as a cellular network consisting of nodes and bonds should not necessarily be understood in a plain bodily sense. One should rather consider it as a representation or emulation of the main characteristics of the physical scenario."* In his article, he refers to earlier works upon which his model was based.

The model described by Requardt is similar in some ways to the link model of space described in this paper. Both models involve the exchange of information between locations in space using links. Nevertheless, the bases upon which the two theories are formulated are different. SMT involves a self-scalar field, the Latent Scalar, whereas Requardt's model is based on the connectivity of line segments in space.

Almost every theory of quantum gravity has a representative geometric object at the small scale. These may be strings, loops, triangles, spin networks, membranes, and/or foams. So what is the characteristic geometric structure of SMT? The fundamental geometric object of SMT is called a smallest link and does not have any well-defined shape. This can be readily seen since the specification of a well-defined shape requires the specification of several independent locations. However, there is only one independent location per smallest link, thus a smallest link does not have any well-defined shape. Each link represents an independent location in space and has an associated number called the Latent Scalar. The Latent Scalar is more fundamental to SMT than any geometric shape.

## 8. CONCLUSION

In conclusion, this article shows how it is possible to construct a mathematical description of space that is formally complete. All physical properties of the multiverse have been shown to be embedded in a self-scalar field called the Latent Scalar, L. This is an important point of progress in people's long journey towards developing a unified theory of physics and Theory of Everything.



## REFERENCES

- C. Beck, *Spatio-Temporal Chaos and Vacuum Fluctuations of Quantized Fields*, (World Scientific 2002).
- C. Callender and N. Huggett, eds., *Physics Meets Philosophy at the Planck Scale: Contemporary Theories in Quantum Gravity*, (Cambridge University Press 2001).
- A. Connes, *Noncommutative Geometry*, (Academic Press 1994).
- W.N. Cottingham and D.A. Greenwood, *An Introduction to the Standard Model of Particle Physics*, (Cambridge University Press 1999).
- A. Einstein, *Relativity: The Special and the General Theory*, translated by R.W. Lawson, (Henry Holt 1920).
- R. Gambini and J Pullin, *Loops, Knots, Gauge Theories and Quantum Gravity*, (Cambridge University Press 2000).
- B. Greene, *The Elegant Universe*, (Vintage Books 2000).
- J. Gribbin, *Schrodinger's Kittens and the Search for Reality*, (Little, Brown & Co. 1996).
- J. Gribbin, *The Search for Superstrings, Symmetry, and the Theory of Everything*, (Little, Brown & Co 1998).
- H.W. Hamber and R.M. Williams, On the measure in simplicial gravity, *Physical Review D* 59 (1999) 064014:1-8.
- H.J. Haubold and A.M. Mathai, Structure of the Universe, *Enc. Applied Physics* 23 (John Wiley & Sons 1998) 47-81.
- M. Jammer, Philosophy of Physics, *Enc. Applied Physics* 13 (John Wiley & Sons 1995) 389-416.
- M. Kaku, *Introduction to Superstrings and M-Theory*, 2<sup>nd</sup> ed., (Springer Verlag 1999).
- M. Kaku, *Strings, Conformal Fields, and M-Theory*, 2<sup>nd</sup> ed., (Springer Verlag 2000).
- D.C. Kay, *Schaum's Outline of Tensor Calculus*, (McGraw-Hill 1988).
- J. Madore, *An Introduction to Noncommutative Differential Geometry and its Physical Applications*, (Cambridge University Press 1999).
- F. Markopoulou, Quantum causal histories, *Classical Quantum Gravity* 17 (2000) 2059-2072.
- F.C. Mish *et al*, eds., *Webster's Ninth New Collegiate Dictionary*, (Meriam-Webster 1988).
- W.K. Panofsky and M. Phillips, *Classical Electricity and Magnetism*, 2<sup>nd</sup> ed., (Addison-Wesley 1962).
- R. Penrose and W. Rindler, *Spinors and Space-Time: Volume 1, Two Spinor Calculus and Relativistic Fields*, (Cambridge University Press 1987).
- R. Penrose and W. Rindler, *Spinors and Space-Time: Volume 2, Spinor and Twistor Methods in Space-Time Geometry*, (Cambridge University Press 1987).
- D.A. Pinnow, *Our Resonant Universe*, online monograph at [www.theoryofeverything.com](http://www.theoryofeverything.com) (2000).
- M.J. Rees, *Our Cosmic Habitat*, (Princeton University Press 2003).
- M. Requardt, (Quantum) spacetime as a statistical geometry of lumps in random networks, *Classical Quantum Gravity* 17 (2000) 2029-2057.
- L. Smolin, *Three Roads to Quantum Gravity*, (Weidenfeld & Nicholson 2000).
- L. Smolin, How far are we from the quantum theory of gravity? preprint [arXiv.org/hep-th/0303185](http://arXiv.org/hep-th/0303185) (2003).